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Introduction to: Fluid Flow Theory

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Introduction

Fluid Mechanics is the study of fluids either in motion (Fluid Dynamics) or at rest (Fluid Statics) and the subsequent effect of the fluid up on the boundaries, which may be either solid surfaces or interfaces with other fluids.

Both gases and liquids are classified as fluids, and the number of fluids engineering application is enormous; breathing, pumping, flaying bodies (air crafts, missiles ...), ships, piping, windmills ... are few.

The essence of the subject – Fluid Flow is a judicious compromise between theory and experiment.

Fluid Mechanics - II devotes for Fluid Dynamics more importantly, Aerodynamics.

Objectives

By the end of this lecture, students will be able to:

- Have a deeper understanding about fluid flow theory
- Determine the velocity profiles of various flow conditions from a simple force balances
- Distinguish various flow types
- Understand the basic laws and governing equations of fluid flow
- Employ differential and integral approaches to formulate/solve a flow phenomena

Reflective Activity

From point of view of Fluid Mechanics, all matter consists of only two states; Fluid and Solid.

Discuss the difference between solid and fluid.

Simple!

A solid can resist shear stress by static deformation, a fluid can not.

Fluid Flow Theory

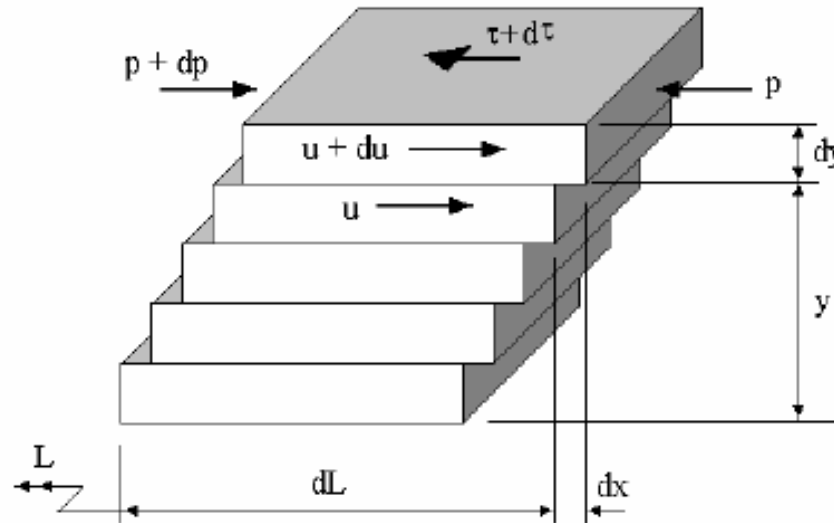
A Fluid

Molecules of fluids exert forces of attraction on each other. In liquids this is strong enough to keep the mass together but not strong enough to keep it rigid. In gases these forces are very weak and cannot hold the mass together.

When a fluid flows over a surface, the layer next to the surface may become attached to it (it wets the surface). The layers of fluid above the surface are moving so there must be shearing taking place between the layers of the fluid.

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Let us suppose that the fluid is flowing over a flat surface in laminated layers from left to right as shown in figure:



y is the distance above the solid surface (no slip surface)

L is an arbitrary distance from a point upstream.

dy is the thickness of each layer.

dL is the length of the layer.

dx is the distance moved by each layer relative to the one below in a corresponding time dt .

u is the velocity of any layer.

du is the increase in velocity between two adjacent layers.

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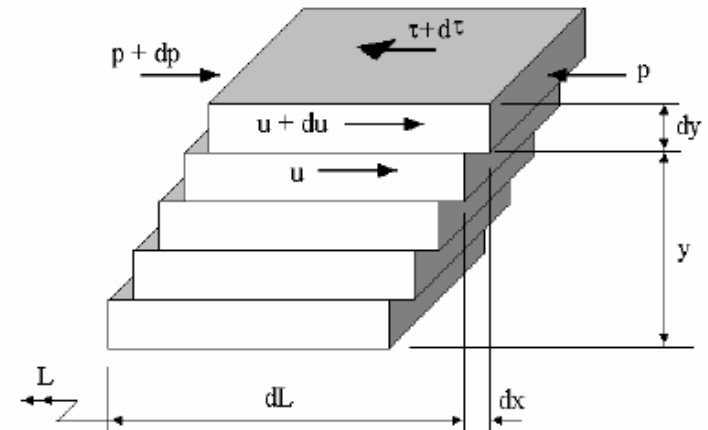
Each layer moves a distance dx in time dt relative to the layer below it. The ratio dx/dt must be the change in velocity between layers so $du = dx/dt$.

When any material is deformed sideways by a (shear) force acting in the same direction, a shear stress τ is produced between the layers and a corresponding shear strain γ is produced. Shear strain is defined as follows.

$$\gamma = \frac{\text{Sideways deformation}}{\text{height of the layer being deformed}} = \frac{dx}{dy}$$

The rate of shear strain is defined as follows.

$$\dot{\gamma} = \frac{\text{shear strain}}{\text{time taken}} = \frac{\gamma}{dt} = \frac{dx}{dt dy} = \frac{du}{dy}$$



Reflective Activity

Discuss the difference between Newtonian and Non – Newtonian Fluids.

It is found that fluids such as water, oil and air, behave in such a manner that the shear stress between layers is directly proportional to the rate of shear strain.

$$\tau = (\text{Constant})\dot{\gamma}$$

Fluids that obey this law are called **NEWTONIAN FLUIDS** and other ways **NON – NEWTONIAN FLUIDS**.

What is that constant?

$$\text{Dynamic Viscosity } (\mu) = \frac{\text{shear stress}}{\text{rate of shear}} = \frac{\tau}{\dot{\gamma}} = \tau \frac{dy}{du}$$

Force Balance and Velocity Distribution – External Flow

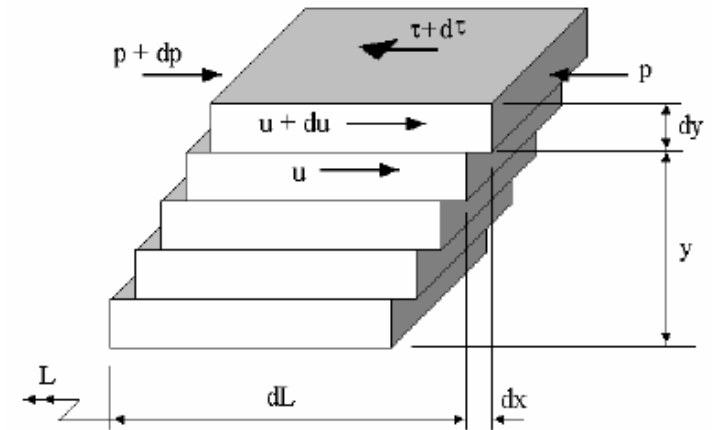
A shear stress τ exists between each layer and this increases by $d\tau$ over each layer. The pressure difference between the downstream end and the upstream end is dp .

The pressure change is needed to overcome the shear stress. The total force on a layer must be zero so balancing forces on one layer (assumed 1 m wide) we get the following.

NB: This is for a flow on flat plate

$$dp \, dy + d\tau \, dL = 0$$

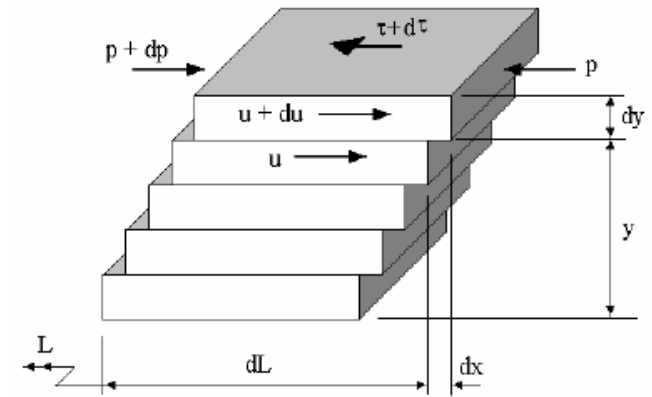
$$\frac{d\tau}{dy} = -\frac{dp}{dL}$$



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It is normally assumed that the pressure declines uniformly with distance downstream so the pressure gradient dp/dL is assumed constant. The minus sign indicates that the pressure falls with distance. Integrating between the no slip surface ($y = 0$) and any height y we get:

$$\begin{aligned} -\frac{dp}{dL} &= \frac{d\tau}{dy} = \frac{d\left(\mu \frac{du}{dy}\right)}{dy} \\ -\frac{dp}{dL} &= \mu \frac{d^2u}{dy^2} \dots\dots\dots(1.1) \end{aligned}$$



Integrating twice to solve u we get the following:

$$\begin{aligned} -y \frac{dp}{dL} &= \mu \frac{du}{dy} + A \\ -\frac{y^2}{2} \frac{dp}{dL} &= \mu u + Ay + B \end{aligned}$$

Cont'd ...

A and B are constants of integration that should be solved based on the known conditions (boundary conditions). For the flat surface considered in figure 1.1 one boundary condition is that $u = 0$ when $y = 0$ (the no slip surface). Substitution reveals the following:

$$0 = 0 + 0 + B \text{ hence } B = 0$$

At some height δ above the surface, the velocity will reach the mainstream velocity u_0 . This gives us the second boundary condition $u = u_0$ when $y = \delta$.

Substituting we find the following:

$$-\frac{\delta^2}{2} \frac{dp}{dL} = \mu u_0 + A\delta$$
$$A = -\frac{\delta}{2} \frac{dp}{dL} - \frac{\mu u_0}{\delta} \text{ hence}$$

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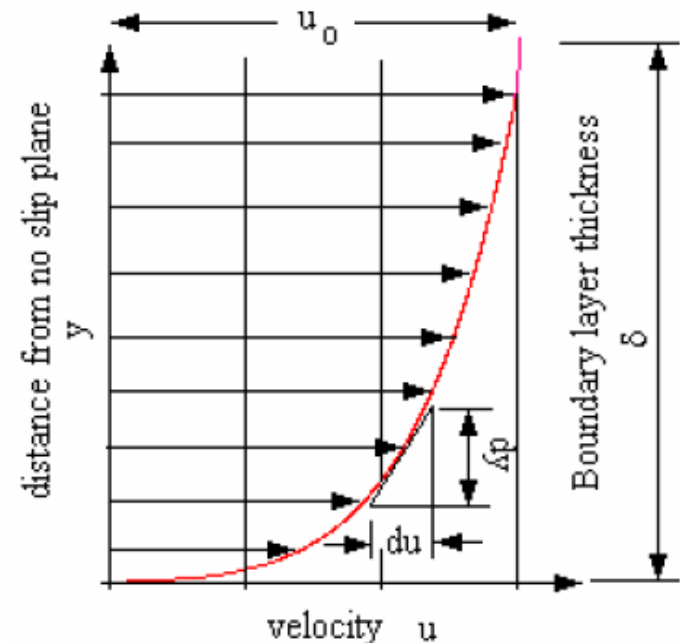
$$-\frac{y^2}{2} \frac{dp}{dL} = \mu u + \left(-\frac{\delta}{2} \frac{dp}{dL} - \frac{\mu u_o}{\delta} \right) y$$

$$u = y \left(\frac{\delta}{2\mu} \frac{dp}{dL} + \frac{u_o}{\delta} \right) \quad \leftarrow \text{The velocity distribution}$$

Plotting u against y gives:

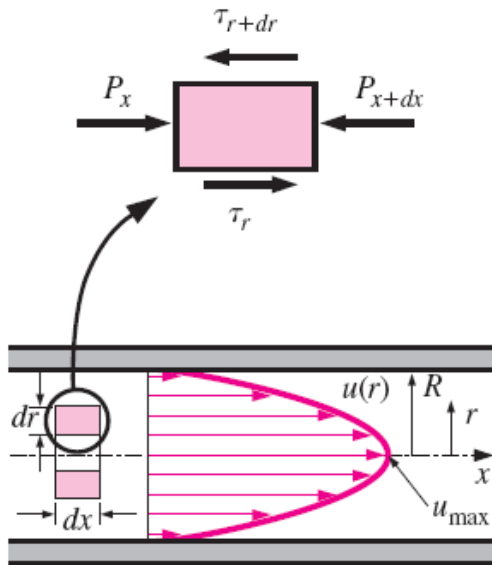
Boundary Layer

The velocity grows from zero at the surface to a maximum at height δ . In theory, the value of δ is infinity but in practice it is taken as the height needed to obtain 99% of the mainstream velocity. This layer is called the boundary layer and δ is the boundary layer thickness. The inverse gradient of the boundary layer is du/dy is the rate of shear strain γ .



Force Balance and Velocity Distribution – Internal Flow

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.



$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

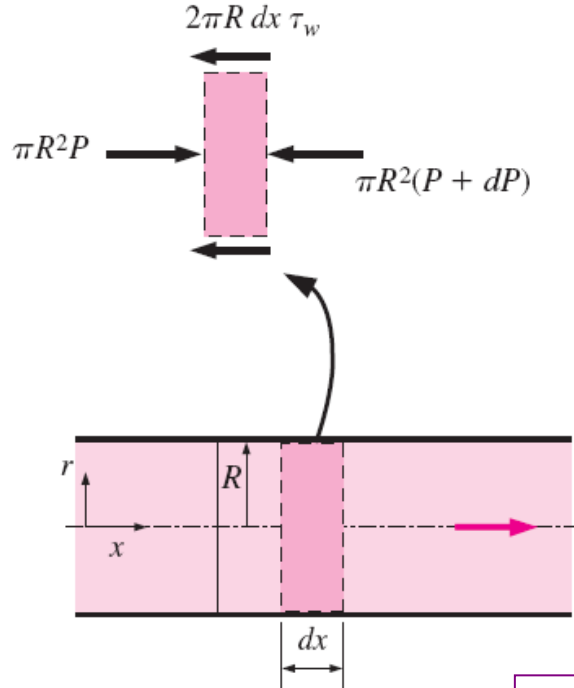
$$r \frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r \frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{du}{dr} \right) = \frac{dP}{dx}$$

Free-body diagram of a ring-shaped differential fluid element of radius r , thickness dr , and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.

Cont'd ...



$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = \frac{r^2}{4\mu} \left(\frac{dP}{dx} \right) + C_1 \ln r + C_2$$

$$\begin{aligned} \partial u / \partial r &= 0 \text{ at } r = 0 && \text{Boundary conditions} \\ u &= 0 \text{ at } r = R \end{aligned}$$

$$u(r) = -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right)$$

Average velocity

$$V_{\text{avg}} = \frac{2}{R^2} \int_0^R u(r) r \, dr = -\frac{2}{R^2} \int_0^R \frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right) r \, dr = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right)$$

Force balance:

$$\pi R^2 P - \pi R^2 (P + dP) - 2\pi R \, dx \, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = 2V_{\text{avg}} \left(1 - \frac{r^2}{R^2} \right) \quad \text{Velocity profile}$$

Free-body diagram of a fluid disk element of radius R and length dx in fully developed laminar flow in a horizontal pipe.

$$u_{\text{max}} = 2V_{\text{avg}}$$

Maximum velocity at centerline

Laminar and Turbulent Flows

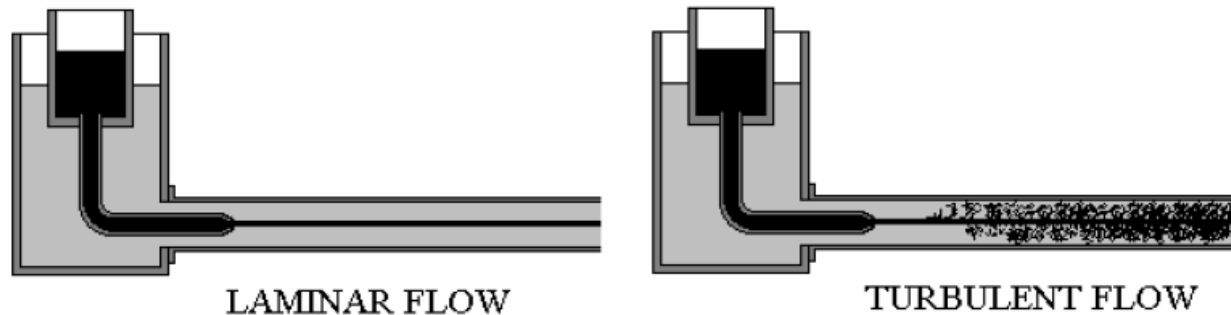
Laminar Flow:

A **stream line** is an imaginary line with no flow normal to it, only along it. When the flow is laminar, the streamlines are parallel and for flow between two parallel surfaces we may consider the flow as made up of parallel laminar layers. In a pipe these laminar layers are cylindrical and may be called **stream tubes**. In laminar flow, no mixing occurs between adjacent layers and it occurs at low average velocities.

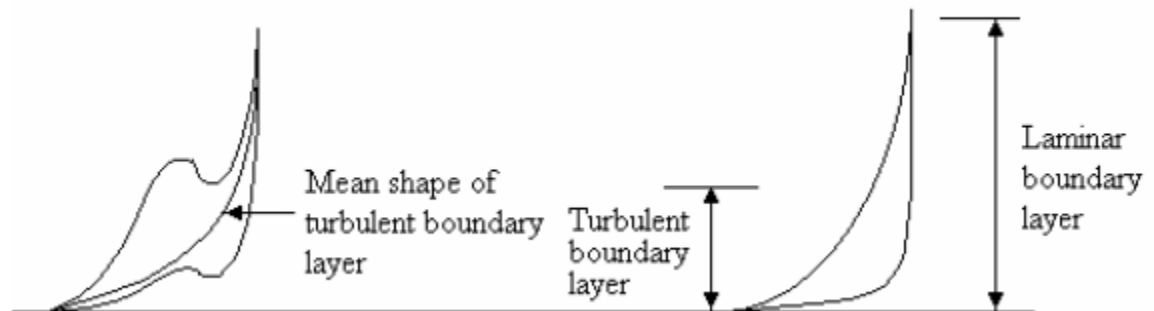
Turbulent Flow:

The shearing process causes energy loss and heating of the fluid. This increases with mean velocity. When a certain critical velocity is exceeded, the streamlines break up and mixing of the fluid occurs. The diagram illustrates Reynolds colored ribbon experiment. Colored dye is injected into a horizontal flow. When the flow is laminar the dye passes along without mixing with the water. When the speed of the flow is increased turbulence sets in and the dye mixes with the surrounding water. One explanation of this transition is that it is necessary to change the pressure loss into other forms of energy such as angular kinetic energy as indicated by small eddies in the flow.

Cont'd ...



It was explained that a **boundary layer** is the layer in which the velocity grows from zero at the wall (no slip surface) to 99% of the maximum and the thickness of the layer is denoted δ . When the flow within the boundary layer becomes turbulent, the shape of the boundary layers waivers and when diagrams are drawn of turbulent boundary layers, the mean shape is usually shown. Comparing a laminar and turbulent boundary layer reveals that the turbulent layer is thinner than the laminar layer. (Why do you think is that?)



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Turbulent flow



Turbulent: Velocity fluctuations and highly disordered motion.



Transition: The flow fluctuates between laminar and turbulent flows.

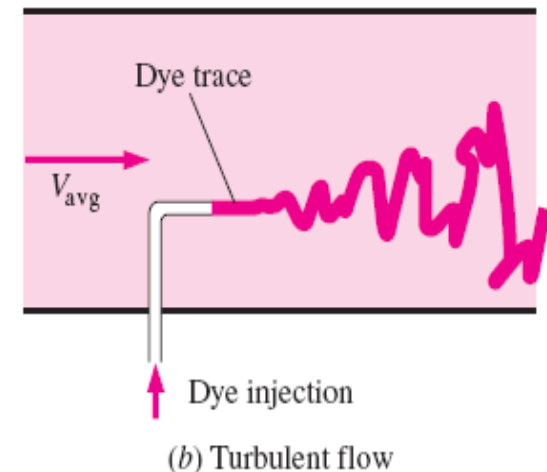
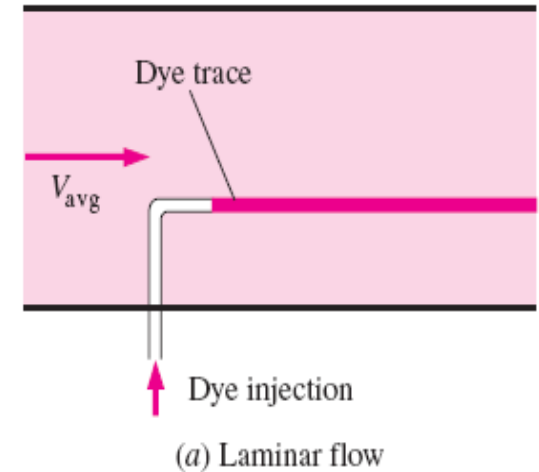


Laminar: smooth streamlines and highly ordered motion.

Most flows encountered in practice are turbulent.

- Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

Laminar and turbulent flow regimes of candle smoke.

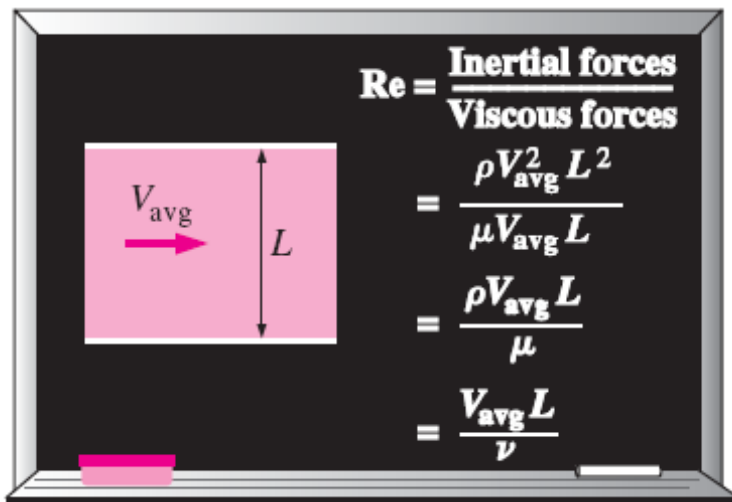


Reynolds Number

The transition from laminar to turbulent flow depends on the **geometry, surface roughness, flow velocity, surface temperature, and type of fluid**.

The flow regime depends mainly on the ratio of **inertial forces** to **viscous forces** (**Reynolds number**).

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$



At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid “in line” (laminar).

Critical Reynolds number, Re_{cr} : The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

Governing Equations of Fluid Flow

Introduction

To formulate/model/solve a fluid flow problem, basic laws will be introduced.

To do so, we might take one of the paths:

Path 1: Seeking to describe the detailed flow pattern at every point (x,y,z) in the field. → **Differential Approach**

Path 2: Working with finite regions, seeking a balance of flow in versus flow out and determining the gross effects such as force, or torque on a body or total energy exchange. → **Integral Approach**

This section has outlined three basic approaches to analysis of arbitrary flow problems:

1. **Integral Approach:** Control volume is finite
2. **Differential (Control volume) Approach:** Control volume is infinitesimally small
3. **Experimental/Dimensional Analysis**

System Vs Control Volume

All the laws of fluid mechanics are written for a **system**, which is defined as an arbitrary quantity of mass of fixed identity. Everything external to this system is denoted by a term **surroundings**, and the system is separated from its surroundings by its **boundaries**.

The laws of mechanics then state what happens when there is an interaction between a system and its surroundings.

Control volume is a finite space in the fluid domain where energy and mass enters/leaves/stores.

Individual masses can be considered as a system where as a control volume must be a specific region.

Basic Laws of Fluid Mechanics

Law 1: First the system is a fixed quantity of mass, denoted by m . Then the mass of a system is conserved and not change.

This law of mechanics has a very simple form, called **conservation of mass**.

$$m_{\text{syst}} = \text{const}$$

$$\frac{dm}{dt} = 0$$

Law 2: If the surrounding exert a net force F on the system, Newton's 2nd law states that the mass begin to accelerate.

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{V}}{dt} = \frac{d}{dt} (m\mathbf{V})$$

This law is applied to a differential control volume of viscous incompressible fluid

Cont'd ...

In fluid mechanics this law Newton's 2nd law is called the **Linear – Momentum Relation**. This law implies three scalar equations.

$$F_x = ma_x$$

$$F_y = ma_y$$

$$F_z = ma_z$$

Law 3: If the surrounding exert a net **moment M** about the center of mass of the system, there will be a **rotation effect**:

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}$$

Where $\mathbf{H} = \sum (\mathbf{r} \times \mathbf{V}) dm$ is the angular momentum of the system about its center of mass.

Cont'd ...

Law 4: If heat dQ is added to the system and work is done by the system the system energy dE must change according to the relation of **first law of thermodynamics**.

$$dQ - dW = dE$$
$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$$

Law 5: **2nd law of thermodynamics** relates entropy change dS to heat added dQ and absolute temperature T .

$$dS \geq \frac{dQ}{T}$$

This law is of less practical importance in incompressible fluid mechanics applications

The Integral Approach

The previous laws or relations are ideal for solid mechanics, where we follow the same system forever because it represents the product we are designing or building.

For example:

- We follow a beam as it deflects under load
- We follow a piston as it oscillates
- We follow a rocket system all its way

But fluid systems do not demand this attention. It is rare that we wish to follow the ultimate path of specific particle of fluid. Instead it is likely that the fluid forms the environment whose effect on the product is needed to be known.

Cont'd ...

We wish to know:

- Wind load on the beam

(In other words, fluid particles in the wind go after they leave the beam is of little interest to the beam designer)

- Fluid pressure on the piston
- Drag and lift load on the rocket

This requires that the basic laws be written to apply to a specific region in the neighborhood of product.

In analyzing control volume, we convert the system laws to apply to a specific region which the system may occupy at any instant of time.

By using integral approach, the basic laws are formulated to apply for this local region called control volume and the flow conditions away from the control volume are irrelevant.

Integral Relations for Control Volume

Conservation of Mass

$$\int_{\text{CV}} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{\text{CS}} \rho(\mathbf{V} \cdot \mathbf{n}) dA = 0$$

The Linear Momentum Equation

$$\sum \mathbf{F} = \frac{d}{dt} \left(\int_{\text{CV}} \mathbf{V} \rho d\mathcal{V} \right) + \int_{\text{CS}} \mathbf{V} \rho(\mathbf{V} \cdot \mathbf{n}) dA$$

The Energy Equation

$$\dot{Q} - \dot{W}_s - (\dot{W}_v)_{ss} = \frac{\partial}{\partial t} \left(\int_{\text{CV}} ep d\mathcal{V} \right) + \int_{\text{CS}} \left(e + \frac{p}{\rho} \right) \rho(\mathbf{V} \cdot \mathbf{n}) dA$$

Differential Relations for a Fluid Particle

Continuity:
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Momentum:
$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p + \nabla \cdot \boldsymbol{\tau}_{ij}$$

Energy:
$$\rho \frac{d\hat{u}}{dt} + p(\nabla \cdot \mathbf{V}) = \nabla \cdot (k \nabla T) + \Phi$$

Thank You

Questions are Welcome!